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ON THE SIMULATION OF A SEISMIC CENTER*

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A qualitative model of the seismic center is proposed in which theoretical "seismograms" resemble observed seismograms with alternating pulse sequence, even in the absence of random signals distortions. In this model the shift development is intermittent leading to a sawtooth shear fracture, so that to each step of the seismogram cooresponds one jump in time and one rectilinear section of the fracture. The kinematic definition of shifts as shear dislocation fractures is used here; in comparison with the force method it makes possible a considerable simplification of mathematical operations, and attainment of more effective results.

The mechanics of the seismic center based on the representation of the center by a brittle shear fracture was dealt with in /1/, where the history of earthquake studies was also presented together with a comprehensive survey of the present state of the theory of tectonic earthquakes.

The assumption that a seismogram in which random distortions of the input signal by all kinds of inhomogeneities, interfaces, etc. have been "washed off" has the form of a smoothly rising curve corresponding to the monotonic development of a single shear crack in one plane /1/ is inadequate.

The qualitative model of the intermittent development of shear is proposed here for the plane case.

1. Description of the earthquake model. Let us assume that the process of breakdown in the seismic center represents the dynamic growth of the edge of some discontinuity surface of the shear components of displacement. Let us also assume that: a) the fracture surface consists of various small flat surfaces as a whole oriented along some main direction, and b) the earthquake is the result of an intermittent (jump-like) growth of the surface of that fracture, with interruptions at the instant of transition of sliding on one small surface to another.

It is reasonable to assume that the main direction coincides with the plane of action of maximum shear stresses at the seismic center.

We make some further assumptions which do not have any fundamental importance but considerably simplify mathematical operations.

1°. The ground outside the surface of fracture is an infinite homogeneous perfectly elastic space.

2°. The physical perturbation field is plane, i.e. independent of one of the space coordinates, and the fracture has the form of various segments of straight lines, with its edge appearing in a plane drawing as a point.

3°. The magnitude and direction of the vector of the displacement jump is specified along the fracture line, i.e. the kinematic definition of fracture applies. It is assumed that only the component of the displacement vector whose direction coincides with the motion of fracture ends becomes discontinuous, while the magnitude of the jump remains constant throughout the process. (It is shown below that the pattern of radiation direction in the case of a single rectilinear dislocation of this type is the same as that of the radiation direction in the case of a rectilinear crack of transverse shear in /2/).

4°. The propagation velocity of the step-like fracture over individual rectlinear sections is constant and does not exceed the limit propagation velocity of the dislocation fracture. (The limit propagation velocity of such fracture represents a certain fraction of the velocity of transverse waves, which depends only on the Poisson's ratio and is determined by properties of the displacement near the fracture edge).

5°. The initial length of the fracture is zero.

2. Construction of solution. We begin by considering the following auxilliary problem. Let us assume that at the initial instant of time $t=0$ a transverse dislocation fracture with a constant displacement jump b begins to propagate at constant velocity v from the coordinate origin in the positive direction of the x-axis. We denote by u_x and u_y the components of the displacement vector along the x- and y-axes, respectively, and by σ_{xx}, σ_{yy} and σ_{xy} the stress tensor components.

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The boundary conditions of this problem (with allowance for its skew-symmetry) are of the form

$$y = 0,\ 0 < x < vt,\ u_x = {}^1/_2\, b,\ \sigma_{yy} = 0, \quad y = 0,\ x < 0,\ x > vt,\ u_x = 0,\ \sigma_{yy} = 0 \tag{2.1}$$

At $t < 0$ the medium was at rest. In the case of the plane problem considered here equations of the dynamic elasticity theory are of the form

$$\frac{\partial u_x^p}{\partial y} = \frac{\partial u_y^p}{\partial x}, \quad \frac{\partial u_x^s}{\partial x} = -\frac{\partial u_y^s}{\partial y}, \quad \Delta u_x^p = \frac{1}{c_p^2}\frac{\partial^2 u_x^p}{\partial t^2}, \quad \Delta u_x^s = \frac{1}{c_s^2}\frac{\partial^2 u_x^s}{\partial t^2}, \quad \Delta u_y^p = \frac{1}{c_p^2}\frac{\partial^2 u_y^p}{\partial t^2}, \quad \Delta u_y^s = \frac{1}{c_s^2}\frac{\partial^2 u_y^s}{\partial t^2} \tag{2.2}$$

$$u_x = u_x^p + u_x^s, \quad u_y = u_y^p + u_y^s \quad (\Delta = \partial^2/\partial x^2 - \partial^2/\partial y^2)$$

where the superscripts p and s denote the longitudinal and transverse displacement components, respectively, and c_p and c_s are velocities of longitudinal and transverse waves, respectively with $(c_p > c_s)$. The longitudinal and transverse waves are subsequently called the P- and S-waves, respectively.

The auxilliary problem formulated above belongs to the class of self-similar problems of the elasticity theory with an $(0, 0)$ self-similarity index. We use the general method for solving such problems /3/ based on the representation of solution of the wave equation in terms of analytic functions of a complex variable, since it enables us to formulate directly the self-similar problem as some Riemann—Hilbert problem for the half-plane (in the simplest case we obtain either the Dirichlet or a mixed problem).

For problems that are skew-symmetric about the x-axis the general solution in terms of the single analytic function W dependent on the complex variables z_p or z_s is of the form

$$u_x = \operatorname{Re}[U_p(z_p) + U_s(z_s)], \quad u_y = \operatorname{Re}[V_p(z_p) + V_s(z_s)] \tag{2.3}$$

$$\sigma_{xx} = \frac{2\mu}{c_s^{-2}} \operatorname{Re}\left\{[c_s^{-2} - 2(c_p^{-2} - z_p^2)]\, W'(z_p)\frac{\partial z_p}{\partial x} + (c_s^{-2} - 2z_s^2)\, W'(z_s)\frac{\partial z_s}{\partial x}\right\}$$

$$\sigma_{yy} = \frac{2\mu}{c_s^{-2}} \operatorname{Re}\left\{(c_s^{-2} - 2z_p^{-2})\, W'(z_p)\frac{\partial z_p}{\partial x} - (c_s^{-2} - 2z_s^2)\, W'(z_s)\frac{\partial z_s}{\partial x}\right\}$$

$$\sigma_{xy} = \frac{\mu}{c_s^{-2}} \operatorname{Re}\left\{4z_p\sqrt{c_p^{-2} - z_p^2}\, W'(z_p)\frac{\partial z_p}{\partial x} + \frac{(c_s^{-2} - 2z_s^2)^2}{z_s\sqrt{c_s^{-2} - z_s^2}}\, W'(z_s)\frac{\partial z_s}{\partial x}\right\}$$

$$\left(z_p = \frac{tx - iy\sqrt{t^2 - c_p^{-2}(x^2 + y^2)}}{x^2 + y^2}, \quad z_s = \frac{tx - iy\sqrt{t^2 - c_s^{-2}(x^2 + y^2)}}{x^2 + y^2}\right)$$

$$U_p'(z) = \frac{2z^2}{c_s^{-2}} W'(z), \quad V_p'(z) = \frac{2z\sqrt{c_p^{-2} - z^2}}{c_s^{-2}} W'(z), \quad U_s'(z) = \frac{c_s^{-2} - 2z^2}{c_s^{-2}} W'(z), \quad V_s'(z) = -\frac{z(c_s^{-2} - 2z^2)}{c_s^{-2}\sqrt{c_s^{-2} - z^2}} W'(z) \tag{2.4}$$

where μ is the shear modulus.

The boundary value problem (2.1), (2.2) can be reduced with the use of formulas (2.3) and (2.4) to the following Dirichlet problem:

$$\operatorname{Im} z = 0, \quad \operatorname{Re} z > v^{-1}, \quad \operatorname{Re} W(z) = {}^1/_2\, b, \quad \operatorname{Im} z = 0, \quad \operatorname{Re} z < v^{-1}, \quad \operatorname{Re} W(z) = 0 \tag{2.5}$$

whose solution is of the form

$$W(z) = -\frac{ib}{2\pi}\ln(1 - vz) \tag{2.6}$$

By substituting (2.6) into (2.5) and (2.3) we obtain for the displacements the following formulas:

$$u_x^p = F_1^p(x, y, t, v, b) = \frac{b\beta^2}{2\pi\gamma^2}\left[2\operatorname{arctg}\frac{\gamma m_p \sin\varphi}{1 - \gamma n_p\cos\varphi} - \gamma m_p(\gamma n_p \sin 2\varphi + 2\sin\varphi)\right] \tag{2.7}$$

$$u_y^p = F_2^p(x, y, t, v, b) = \frac{b\beta^2}{2\pi\gamma^2}\Bigg[(2 - \gamma^2)\ln(n_p + m_p) + \gamma m_p(\gamma n_p\cos 2\varphi + 2\cos\varphi) +$$

$$\sqrt{1 - \gamma^2}\ln\frac{\sin^2\varphi\,(n_p\sqrt{1 - \gamma^2} - m_p)^2 + [\gamma - \cos\varphi\,(n_p - \sqrt{1 - \gamma^2}m_p)]^2}{(1 - \gamma n_p\cos\varphi)^2 + \gamma^2 m_p^2\sin^2\varphi}\Bigg]$$

$$u_x^s = F_1^s(x, y, t, v, b) = \frac{b\beta^2}{2\pi\gamma^2}\left[\left(\frac{\gamma^2}{\beta^2} - 2\right)\operatorname{arctg}\frac{\gamma m_s\sin\varphi}{\beta - \gamma n_s\cos\varphi} + \frac{\gamma m_s}{\beta^2}(\gamma n_s\sin 2\varphi + 2\beta\sin\varphi)\right]$$

$$u_y^s = F_2^s(x, y, t, v, b) = -\frac{b\beta^2}{2\pi\gamma^2}\Bigg[2\ln(n_s + m_s) + \frac{\gamma m_s}{\beta^2}(\gamma n_s\cos 2\varphi + 2\beta\cos\varphi) +$$

$$\frac{2\beta^2 - \gamma^2}{2\beta\sqrt{\beta^2 - \gamma^2}}\ln \times \frac{\sin^2\varphi\,(n_s\sqrt{\beta^2 - \gamma^2} - \beta m_s)^2 + [\gamma - \cos\varphi(\beta n_s - m_s\sqrt{\beta^2 - \gamma^2})]^2}{(\beta - \gamma n_s\cos\varphi)^2 + \gamma^2 m_s^2\sin^2\varphi}\Bigg]$$

$$r=\sqrt{x^2+y^2},\quad \varphi=\operatorname{arctg}\frac{y}{x},\quad \beta=\frac{c_s}{c_p},\quad \gamma=\frac{v}{c_p},\quad n_p=c_p\frac{t}{r},\quad n_s=c_s\frac{t}{r},\quad m_p=\sqrt{n_p^2-1},\quad m_s=\sqrt{n_s^2-1}$$

Functions F_1^p, F_1^s, F_2^p and F_2^s represent fundamental solutions which, by the principle of superposition, enable us to construct readily a solution of the problem for any arbitrary curvilinear dislocation fracture propagating at an arbitrary variable velocity $v(t)$ and with a displacement jump defined by an arbitrary function of time and coordinates. For the purpose of this investigation it is sufficient to solve the problem of intermittent propagation of the sawtooth dislocation fracture using the model defined in Sect.1 by the assumptions 1°—5°.

Let at the instant of time $t=t_0$ a transverse dislocation fracture with constant displacement jump b_1 begin to propagate at constant velocity v_1 from point (x_0, y_0) along a line at angle α_0 to the x-axis. At some instant $t=t_1$ the fracture tip stops, then at instant $t=t_2$ its motion resumes along a line at angle α_1 to the x-axis at some velocity v_2 with displacement jump b_2; this motion continues up to the instant of time $t=t_3$ when the second interruption of fracture propagation takes place. At any of the subsequent instants of time $t=t_{2k}$, the fracture tip resumes its motion along a line at angle α_k to the x-axis at some constant velocity v_{k+1} and with displacement jump b_{k+1}; then, at instant of time $t=t_{2k+1}$ its propagation ceases.

The solution of this problem consists of the superposition of the previously derived fundamental solution, with the obvious substitution of arguments

$$x\to\bar{x}_k,\quad y\to\bar{y}_k,\quad t\to t-t_k,\quad v\to v_k,\quad b\to b_k$$

where

$$\bar{x}_{2k}=(x-x_k)\cos\alpha_k+(y-y_k)\sin\alpha_k,\quad \bar{x}_{2k-1}=(x-x_k)\cos\alpha_{k-1}+(y-y_k)\sin\alpha_{k-1}$$

$$\bar{y}_{2k}=-(x-x_k)\sin\alpha_k+(y-y_k)\cos\alpha_k,\quad \bar{y}_{2k-1}=-(x-x_k)\sin\alpha_{k-1}+(y-y_k)\cos\alpha_{k-1}$$

$$x_k=x_0+\sum_{l=1}^{k}v_l\,(t_{2l-1}-t_{2l-2})\cos\alpha_{l-1},\quad y_k=y_0+\sum_{l=1}^{k}v_l\,(t_{2l-1}-t_{2l-2})\sin\alpha_{l-1}$$

For a finite number n of jumps this solution can be represented in the following analytic form:

$$u=\sum_{k=0}^{n-1}F(\bar{x}_{2k},\bar{y}_{2k},t-t_{2k},v_{k+1},b_{k+1})-\sum_{k=0}^{n-1}F(\bar{x}_{2k+1},\bar{y}_{2k+1},t-t_{2k+1},v_{k+1},b_{k+1})\tag{2.8}$$

where the superscript and subscript at displacement u are the same as in the fundamental solution F defined by formulas (2.7); these indices have been omitted here for simplicity. Note that the case of arbitrary specification of displacement jump, fracture trajectory and its propagation velocity can be from (2.8) by passing in it to the limit $n\to\infty$, $t_{k+1}-t_k\to 0$, $v_{k+1}-v_k\to 0$, $b_{k+1}-b_k\to 0$, and $\alpha_{k+1}-\alpha_k\to 0$.

Similar problems in force formulation, i.e. by specifying the law of interaction between opposite fracture edges are not amenable to effective investigation.

3. Analysis of solution and numerical computations.

For seismological applications of greatest interest is the asymptotic behavior of solutions at considerable distances from the perturbation source at the first arrival of P- and S-waves at the observation point (precursor asymptotics). This solution is derived from the fundamental solution (2.7) by passing to the limit $n_p\to 1$ and $n_s\to 1$. The asymptotic formulas for the magnitude of the displacement vector in P- and S-waves we have

$$|u^p|=\sqrt{(u_x^p)^2+(u_y^p)^2}=b\frac{\gamma\beta^2}{2\pi}\frac{|\sin 2\varphi|}{1-\gamma\cos\varphi}\lambda_p,\quad |u^s|=\sqrt{(u_x^s)^2+(u_y^s)^2}=b\frac{\gamma}{2\pi}\frac{|\cos 2\varphi|}{\beta-\gamma\cos\varphi}\lambda_s\tag{3.1}$$

$$\lambda_p=\begin{cases}m_p, & n_p>1\\ 0, & n_p<1,\end{cases}\qquad \lambda_s=\begin{cases}m_s, & n_s>1\\ 0, & n_s<1\end{cases}$$

For $0<\varphi<{}^1/_2\pi$ and $\pi<\varphi<{}^3/_2\pi$ the direction of the displacement vector $\mathbf{u}^p$ asymptotically approaches the direction of the radius vector drawn from the source to the observation point, while for ${}^1/_2\pi<\varphi<\pi$ and ${}^3/_2\pi<\varphi<2\pi$ it approaches the vector line in the opposite direction. For $-{}^1/_4\pi<\varphi<{}^1/_4\pi$ and ${}^3/_4\pi<\varphi<{}^5/_4\pi$ the direction of the shift vector $\mathbf{u}^s$ asymptotically approaches that radius-vector, and for ${}^1/_4\pi<\varphi<{}^3/_4\pi$ and ${}^5/_4\pi<\varphi<{}^7/_4\pi$ it approaches the vector line in the opposite direction. The plane in which the respective displacement vector vanishes is called nodal. One of the nodal planes for P-waves evidently coincides with the direction of fracture propagation and the second is normal to it, for S-waves the nodal planes are at $\pm\pi/_4$ angles to the direction of fracture propagation. The dependence of displacement vectors $|u^p|$ and $|u^s|$ on angle φ is called the radiation direction diagram. It can be used for determining the directions of vectors $\mathbf{u}^p$ and $\mathbf{u}^s$, taking into account the indicated sign rule.

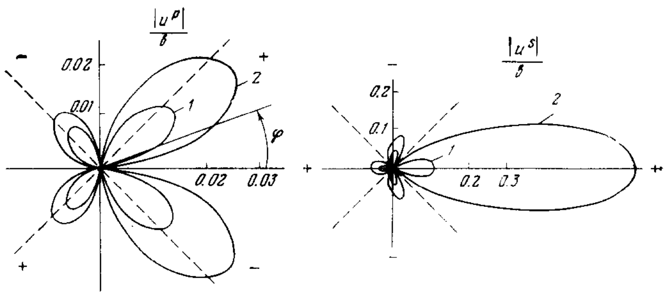

Fig.1

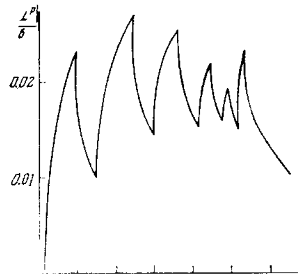

Fig.2

Radiation direction diagrams for P- and S-waves are shown in Fig.1 for two different fracture propagation velocities (curves *1* and *2* correspond to velocities v equal $0.6\,c_s$ and $0.9\,c_s$). These curves, calculated by formulas (3.1), show the substantial dependence of radiation intensity and direction on fracture propagation velocity.

For low velocities of discontinuity propagation in P-waves, the radiation maxima are directed along the bisectrices, and for the S-waves along the fracture propagation direction and along the normal to it. As the fracture propagation velocity increases, the importance of the denominator in formulas (3.1) also increases, which leads to the asymmetry of the radiation direction relative to the plane normal to discontinuities in the P- and S-waves, as well as the asymmetry of that direction relative to the bisecting planes in P-waves. Thus for $v = 0.9\,c_s$ the two highest maxima occur along lines at angles $\pm 36^\circ$ to the x-axis, while the two lowest maxima are along the line at $\pm 128^\circ$ angle to the x-axis. As the fracture velocity approaches c_s, a considerable increase of the radiation maximum occurs in S-waves in the direction of propagation of the fracture end; simultaneously the maxima undergo an angular displacement. Thus for $v = 0.9\,c_s$ such maxima occur along lines at $\pm 75^\circ$ angles to the x-axis. It should be pointed out that the radiation maximum in S-waves is by one order /of magnitude/ higher than in P-waves. Owing to this the basic part of energy radiates along the fracture plane, while the previously indicated asymmetry of radiation direction makes it possible to estimate the fracture propagation velocity and can, also, be used for a unique selection of the fracture plane.

Use of the fundamental solution asymptotics together with the diagram of direction radiation enables us to determine by superposition the dependence of the displacement on time (the theoretical seismogram) at any observation point. One of such seismograms for P-waves appears in Fig.2 for the case of intermittent propagation of a rectilinear fracture from the coordinate origin in the positive direction of the x-axis.

The curve was determined for the case of six jumps and the following values of parameters:

$$c_p = 7.5\,\text{km/s},\ c_s = 4.5\,\text{km/s},\ r = 1000\,\text{km},\ \varphi = 40^\circ$$
$$v_1 = 0.9\,c_s,\ v_2 = 0.8\,c_s,\ v_3 = 0.7\,c_s,\ v_4 = 0.6\,c_s$$
$$v_5 = 0.5\,c_s,\ v_6 = 0.9\,c_s,\ t_0 = 0,\ t_1 = 3\,\text{s},\ t_2 = 4\,\text{s}$$
$$t_3 = 7\,\text{s},\ t_4 = 8\,\text{s},\ t_5 = 10\,\text{s},\ t_6 = 11\,\text{s},\ t_7 = 12\,\text{s}$$
$$t_8 = 12.5\,\text{s},\ t_9 = 13\,\text{s},\ t_{10} = 13.5\,\text{s},\ t_{11} = 14\,\text{s},\ \alpha_k = 0,\ b_k = b$$

The curve maxima in Fig.2 correspond to the instants of arrival of waves arising at stoppages of the fracture tip, and the minima relate to the instants of arrival of waves generated at the resumption of propagation of the dislocation fracture tip. Thus the intermittent motion of the fracture tip, i.e. when the fracture tip propagation velocity vanishes for some time intervals, produces the characteristic sawtooth curve. When the fracture propagation velocity is smooth, the pattern of displacements in P-and S-waves is, obviously, of the form of a smoothly varying curve; the case in which there are time intervals in which the fracture propagation velocity is zero is characterized by a sawtooth curve of the type shown in Fig.2.

The alternating sequence of pulses, a characteristic of the majority of recorded seismograms, is difficult to explain if a rectilinear fracture propagation is assumed. However theoretical seismograms with sequencies of alternating pulses are, as a rule, obtained in the case of intermittent curvilinear fracture propagation. To explain this feature we shall, first, consider the case of a single change of the propagation direction from the initial to one at angle α to it. We assume the observation point Q at which the P-pulse is recorded to be at considerable distance from the fracture tip, and to lie on a line at angle $\alpha/2$ to the x-axis. For definiteness we assume that $0 < \alpha < \pi/2$. Then, as can be seen from the direction diagrams in Fig.1, first, a positive pulse reaches point Q. After the fracture had turned by an angle α, negative pulses begin to arrive at the observation point Q, because the radiation direction diagram turns simultaneously with the fracture by the angle α.

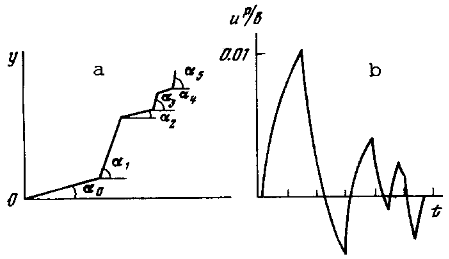

Fig.3

With several changes of direction of the dislocation fracture about some main propagation line (as defined in the general model proposed in Sect.1), theoretical seismograms with alternate sign sequences of pulses are obtained in which a single step of the seismogram corresponds to each rectilinear section of fracture.

As an example, the case of a stepwise fracture with six rectilinear sections shown in Fig.3,a was calculated. The physical parameters used in this case were exactly the same as in the previously considered example of rectilinear intermittent fracture propagation (see Fig.2 and formulas (3.2)), and the direction changes of the fracture were specified as follows: $\alpha_0 = 10°$, $\alpha_1 = 70°$, $\alpha_2 = 20°$, $\alpha_3 = 60°$, $\alpha_4 = 15°$ and $\alpha_5 = 65°$. The obtained seismogram in P-waves shown in Fig.3b, resembles actual seismograms.

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